

Algebra

PENG

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1 Introduction

Definition 1.1 (mapping). *Let A, B be sets.*

$$f : A \rightarrow B \quad (1.1)$$

Definition 1.2 (f).

$$x \mapsto f(x) \quad (1.2)$$

- *injective* : $x \neq y \Rightarrow f(x) \neq f(y)$
- *surjective* : $\forall b \in B \Rightarrow \exists a \in A, f(a) = b$
- *bijective* : both surjective and injective

Definition 1.3 (composite map $g \circ f$).

$$(g \circ f)(x) = g(f(x)) \quad (1.3)$$

Definition 1.4 (commutative). *Composite maps are equal.*

$$\begin{aligned} A_1 &\xrightarrow{f_1} A_2 \xrightarrow{f_2} \dots \xrightarrow{f_{n-1}} A_n \\ A_1 &\xrightarrow{g_1} B_2 \xrightarrow{g_2} \dots \xrightarrow{g_{m-1}} B_m = A_n \\ f_{n-1} \circ \dots \circ f_1 &= g_{m-1} \circ \dots \circ g_1 \end{aligned}$$

2 Groups

2.1 Monoids

Definition 2.1 (law of composition). *Let S be a set:*

$$S \times S \rightarrow S \quad (2.1)$$

Definition 2.2 (monoid).