Analysis

PENG

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1 Numbers

1.1 Natural Number system

- This is how mathematics works - it treats its objects abstractly, caring only about what properties the objects have, not what the objects are or what they mean [1].
- It is pointless, at least from a mathematician's standpoint, as to argue about which model is the true one; as long as it obeys all the axioms and does all the right things, that's good enough to do maths.

Axiom 1.1. *Peano axioms:*

1. *0 is a natural number.*
2. *If n is a natural number, then $n{+}{+}$ is also a natural number.*
3. $n{+}{+} \neq 0$
4. $n \neq m \Rightarrow n{+}{+} \neq m{+}{+}$
   $n{+}{+} = m{+}{+} \Rightarrow n = m$
5. *Principle of mathematical induction:*
   Suppose that $P(0)$ is true, and suppose that whenever $P(n)$ is true, $P(n{+}{+})$ is also true. Then $P(n)$ is true for every natural number n.

Definition 1.2. *Natural number system $\mathbb{N}$:*

$$\begin{aligned} 1 &:= 0{+}{+} \\ 2 &:= 1{+}{+} \\ 3 &:= 2{+}{+} \\ &\vdots \end{aligned}$$

Definition 1.3. *Addition:*
Let m be a natural number.

$$\begin{aligned} 0 + m &:= m \\ (n{+}{+}) + m &:= (n + m){+}{+} \end{aligned}$$

Lemma 1.4. *For any natural number n, $n + 0 = n$.*

Proof. $0 + 0 = 0$
Suppose $n + 0 = n$
$(n{+}{+}) + 0 = (n + 0){+}{+} = n{+}{+}$ $\square$

Lemma 1.5. *For any natural number n and m, $n + (m{+}{+}) = (n + m){+}{+}$.*

Proof. $\square$

References

[1] Terence Tao. *Analysis I*. Vol. 1. Springer, 2022. ISBN: 978-981-19-7261-4.