Machine Learning

PENG

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Contents

1 Linear Algebra

Definition 1.1. *Linear dependency:*

$$A_{n\times n}x_n = 0$$
$$\begin{bmatrix} \vdots \\ a_1 \\ \vdots \end{bmatrix} x_1 + \begin{bmatrix} \vdots \\ a_2 \\ \vdots \end{bmatrix} x_2 + \begin{bmatrix} \vdots \\ a_n \\ \vdots \end{bmatrix} x_n = 0$$

If $a_1, \ldots, a_n$ is linearly independent $\Rightarrow x_1 = \cdots = x_n = 0$
If $a_1, \ldots, a_n$ is linearly dependent $\Leftrightarrow det(A) = 0 \Rightarrow$ nonzero solution

Definition 1.2. *Eigenvalue:*

$$A_{n\times n}x_n = \lambda x_{n\times 1} \tag{1.1}$$
$$(A - \lambda I)x = 0$$
$$|A - \lambda I| = 0 \tag{1.2}$$

Definition 1.3. *Orthogonal:*

2 Singular Value Decomposition

Definition 2.1. *SVD [1]:*

$$X_{n\times m} = U_{n\times n}\Sigma_{n\times m}V^T_{m\times m} \tag{2.1}$$

where
$UU^T = U^TU = I_{n\times n}$
$VV^T = V^TV = I_{m\times m}$
$$\Sigma_{n\times m} = \begin{bmatrix} \lambda_1 > 0 & & \\ & \ddots & \\ & & \lambda_m > \lambda_{m-1} \\ & \vdots & \end{bmatrix}$$

Definition 2.2. *Economy SVD ($n \geq m$):*

$$\begin{aligned} X_{n\times m} &= \begin{bmatrix} \hat{U}_{n\times m} & \hat{U}^{\perp}_{n\times(n-m)} \end{bmatrix} \begin{bmatrix} \hat{\Sigma}_{m\times m} \\ 0_{(n-m)\times m} \end{bmatrix} V^T_{m\times m} \\ &= \hat{U}_{n\times m}\hat{\Sigma}_{m\times m}V^T_{m\times m} \end{aligned} \tag{2.2}$$

References

[1] Steven L Brunton and J Nathan Kutz. *Data-driven science and engineering: Machine learning, dynamical systems, and control*. Cambridge University Press, 2022. ISBN: 1009098489.